

Philadelphia University

Electrical Engineering Department

Electrical Machines

Transformers

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The transformer consists of a laminated iron core about which are wound two or more sets of windings. Voltage is supplied to one set of windings, called the primary, which builds up a magnetic flux through the iron. This flux generates a counter electromotive force in the primary which limits the current drawn from the supply. The same flux system generates an electromotive force in the second winding, or secondary, to which the load is connected. This secondary electromotive force causes current to be supplied to the load. Hence power is transferred electromagnetically from the primary to the secondary winding which supplies it electrically to the load. Little power is lost in such a transformation. Efficiencies may range from 97 to 99 %.

1. Electromotive - force Equation. When an electric circuit is supplied with a voltage, that circuit must take sufficient current from the source to build up a counter electromotive force equal and opposite to the voltage applied.

The primary of the transformer is a circuit wound on an iron core. The resistance of the winding is purposely made low. Thus the counter electromotive is made up essentially of a voltage produced by the action of the flux on the primary turns. The instantaneous ^{value} of this counter e.m.f. is

$$e_1 = -N_1 \frac{d\phi}{dt} \quad \text{Volts.} \quad (1)$$

where N_1 is the number of turns on the primary winding and ϕ is the instantaneous value of the flux in the core. Assuming the primary resistance to be zero, the applied voltage v_1 is equal and opposite to e_1 , or

$$v_1 = N_1 \frac{d\phi}{dt} \quad (2)$$

If the applied voltage is sinusoidal, that is,

$$v_1 = V_{\max} \sin 2\pi f t, \text{ then } \phi = \phi_{\max} \sin 2\pi f t. \text{ and}$$

$$e_1 = -N_1 \frac{d}{dt} [\phi_{\max} \sin 2\pi f t]$$

$$= -N_1 \phi_{\max} (2\pi f) \cos 2\pi f t = -E_{\max} \cos 2\pi f t$$

where $E_{\max} = 2\pi N_1 f \phi_{\max}$.

$$\& E_{1,r.m.s} = E_1 = \frac{2\pi N_1 f \phi_{\max}}{\sqrt{2}} = 4.44 N_1 f \phi_{\max} \dots (3)$$

which is known as the e.m.f equation.

Thus if v_1 and e_1 are sinusoidal, the flux wave must be cosinusoidal. These quantities can be represented as vectors as shown in Fig. 1.

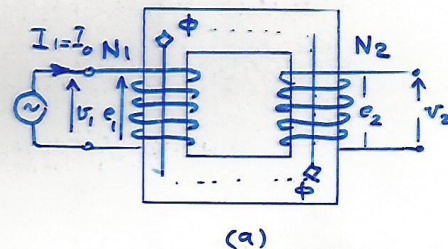
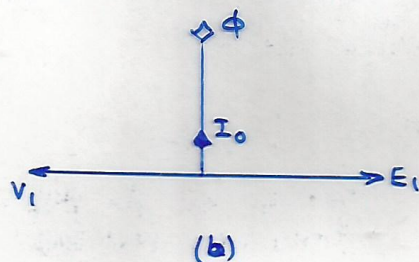


Fig. 1

I_0 = no load or exciting current.

Fig. 1. represents an ideal transformer. The IDEAL transformer has the following characteristics:

- (i) Zero winding resistance
- (ii) Zero leakage flux, i.e. all the flux produced by one winding linked by the other winding
- (iii) Iron core is lossless, i.e. no hysteresis and eddy current losses.

For the above assumptions:

$$E_1 = 4.44 \phi_m f N_1$$

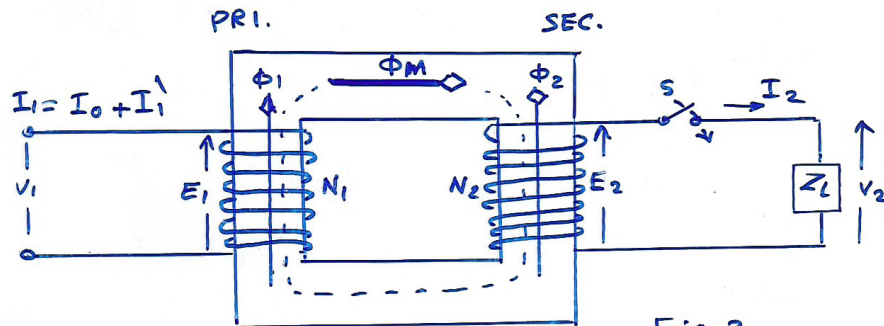
$$E_2 = 4.44 \phi_m f N_2$$

Hence

$$\boxed{\frac{E_1}{E_2} = \frac{N_1}{N_2}} \dots (4)$$

2. IDEAL TRANSFORMER ON LOAD

The theory of the ideal transformer on load can be easily understood from Fig. 2.



The steps of operation of an ideal transformer are as follows:

- A sinusoidal voltage V_1 applied to the primary causes current I_0 to flow at no load (S is open)
- I_0 creates a sinusoidal flux Φ_M in the ferromagnetic core.
- Φ_M : (1) induce a voltage $E_1 = N_1 \frac{d\Phi_M}{dt} \Rightarrow E_1$
 E_1 opposes V_1 and thereby limits I_0 .
 (2) induce a voltage $E_2 = N_2 \frac{d\Phi_M}{dt} \Rightarrow E_2$
 in the secondary winding.
- Closing S, current I_2 will flow through the secondary winding.
- I_2 creates Φ_2 in opposition to Φ_M . , the core flux will be $(\Phi_M - \Phi_2)$, this will reduce the total flux in the core causing E_1 to drop. As a result, primary current will increase by increment I_1' .
- I_1' will produce Φ_1 opposes and neutralize Φ_2 , leaving Φ_M as the main flux to maintain E_1 and E_2 constant. Hence $MMF_1 = MMF_2$.
 or $I_1 N_1 = I_2 N_2$ ----- (5)

Hence

$$\boxed{\frac{I_1}{I_2} = \frac{N_2}{N_1}} \quad \text{----- (6)}$$

However Eqs. 4 & 6 can be given in details as:

$$V_1 = \frac{N_1}{N_2} V_2 \quad \text{and} \quad V_2 = \frac{N_2}{N_1} V_1$$

$$I_1 = \frac{N_2}{N_1} I_2 \quad \text{and} \quad I_2 = \frac{N_1}{N_2} I_1$$

To sum up, in an ideal transformer voltages are transformed in direct ratio of turns, but currents in the inverse ratio.

The ratio of transformation $\frac{N_1}{N_2}$ is usually called a .

$$\therefore a = \frac{N_1}{N_2} = \text{Turns ratio of the transformer.}$$

If the load is purely resistive R_L , then from the above equations:

$$\begin{array}{l|l} V_2 = I_2 R_L & \frac{V_1}{I_1} = \left(\frac{N_1}{N_2}\right) V_2 / \left(\frac{N_2}{N_1}\right) I_2 \\ \hline \therefore R_L = \frac{V_2}{I_2} & = \left(\frac{N_1}{N_2}\right)^2 \frac{V_2}{I_2} = a^2 R_L. \end{array}$$

Consequently, as far as the effect is concerned, a load resistor R_L in the secondary circuit may be replaced by an equivalent resistor R'_L in the primary circuit, provided that

$$R'_L = \frac{V_1}{I_1} = a^2 R_L \quad \& \quad \frac{Z_1}{Z_2} = \left(\frac{N_1}{N_2}\right)^2 = a^2.$$

Note: Such analysis holds also for impedances.

$$\text{Power in the primary} = I_1 V_1 = \frac{N_1}{N_2} V_2 \times \frac{N_2}{N_1} I_2 = V_2 I_2$$

$$\therefore \text{power in the primary} = \text{power in the secondary.}$$

Hence in an ideal transformer, impedances are transformed in the direct ratio squared, and power and volt-ampere are unchanged.

Example 1: The voltage $v = 100 \sin 377t - 20 \sin 1885t$, is applied to a 200 - turn transformer winding. Derive the equation for the flux in the core, neglecting leakage flux and winding resistance. Determine the rms values of the voltage and the flux.

Solution:

$$e_1 = v_1 = -N_1 \frac{d\phi}{dt}$$

$$\begin{aligned} \therefore \phi &= -\frac{1}{N_1} \int v_1 dt = -\frac{1}{200} \left[\int 100 \sin 377t dt + \int -20 \sin 1885t dt \right] \\ &= -\frac{1}{200} \left[-\frac{100}{377} \cos 377t + \frac{20}{1885} \cos 1885t \right] \\ &= 0.00133 \cos 377t - 5.3 \times 10^{-5} \cos 1885t \end{aligned}$$

The r.m.s values are:

$$\Phi = \sqrt{\left(\frac{0.00133}{\sqrt{2}}\right)^2 + \left(\frac{5.3 \times 10^{-5}}{\sqrt{2}}\right)^2} = 9.4 \text{ mWb.}$$

$$V = \sqrt{\left(\frac{100}{\sqrt{2}}\right)^2 + \left(\frac{20}{\sqrt{2}}\right)^2} = 72.1 \text{ Volts.}$$

Example 2: A transformer is rated 100 kVA, 400V primary and 200V secondary voltages. Assume the transformer is ideal, determine the turns ratio and the winding current ratings.

Solution:

$$\text{The turns ratio } a = \frac{E_1}{E_2} = \frac{V_1}{V_2} = \frac{400}{200} = 2.$$

$$\text{The primary current } I_1 = \frac{100 \times 10^3}{400} = 250 \text{ A}$$

$$\text{The secondary current } I_2 = \frac{100 \times 10^3}{200} = 500 \text{ A.}$$

Note: The transformer ratings are always given in terms of RMS parameters.

Example .3: A 100-KVA transformer has 400 turns on the primary and 80 turns on the secondary. The primary and secondary resistances are 0.3Ω and 0.01Ω respectively, and the corresponding leakage reactances are 1.1Ω and 0.035Ω respectively. The supply voltage is 2200V. Calculate:
 (a) the equivalent impedance referred to the primary circuit.
 (b) the primary and secondary currents on rated-load referred to the primary.

Solution:-

(a) transformation ratio (turns ratio) $a = \frac{N_1}{N_2} = \frac{400}{80} = 5$

equivalent resistance referred to primary $\{ = R_{eq} = R_1 + R_2 a^2$
 $= 0.3 + 0.01 (5)^2 = 0.55\Omega$

equivalent reactance referred to primary $\{ = X_{eq} = 1.1 + 0.035 (5)^2$
 $= X_1 + X_2 a^2 = 1.975\Omega$

equivalent impedance referred to primary $\{ = Z_{eq} = \sqrt{R_{eq}^2 + X_{eq}^2}$
 $= \sqrt{(0.55)^2 + (1.975)^2}$
 $= 2.05\Omega$

(b) Rated load primary current $= \frac{\text{Rated VA}}{\text{primary voltage}}$
 $= \frac{100 \times 10^3}{2200} = 45.45A$

secondary current $I_2 = a I_1 = 5 \times 45.45 = 227.25A$

Actual Transformer

7/2

Transformer Reactances and Equivalent Circuits

The actual transformer analyses must take into account the effects of winding resistances, magnetic leakage and exciting current.

1. The no-load current: The no load current I_0 of an actual transformer composed of the exciting current I_{ex} which can be resolved into core-loss component I_c in phase with the counter e.m.f E_1 and a magnetizing component I_m lagging E_1 by 90° .

The core-loss current component I_c produced by hysteresis core loss component and eddy current loss component. The equivalent circuit (Fig. 4) depicts the effect of I_c and I_m . In this circuit the equivalent sinusoidal exciting current I_{ex} is accounted for by means of a shunt branch connected across E_1 , comprising a noninductive resistance R_c in parallel with a lossless inductance X_m .

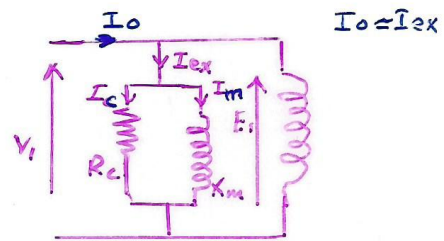


Fig. 4

2. Effects of resistance and leakage fluxes.

The primary winding of an actual transformer comprises of a series resistance and inductance. Hence a series resistance and reactance combination can be connected across E_1 , as shown in Fig. 5

where R_1 is accounted for the copper loss or IR drop in the primary winding, and X_1 is the leakage reactance of the primary winding (due

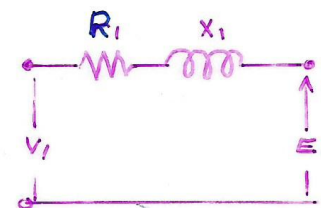


Fig. 5

to the flux that cuts only the primary winding Φ_{L1} . ($\Phi_m \Rightarrow \Phi_{L1} + \Phi_{L2} + \Phi_{12}$). This produces a voltage $+ \Phi_{21}$

drop $I X$ caused by X_1 in the primary circuit.

When the transformer is loaded a secondary current I_2 will flow in the secondary winding causing IR and IX drop

due to the secondary winding resistance R_2 and leakage

reactance X_2 (due to ϕ_{L2} as shown in Fig. 6(a).

Fig. 6(b) shows schematically how the primary and secondary voltage drops are represented due to resistance and leakage reactances.

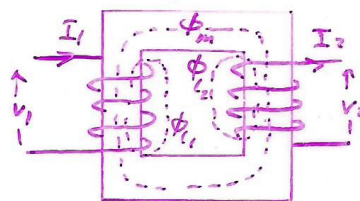


Fig. 6(a)

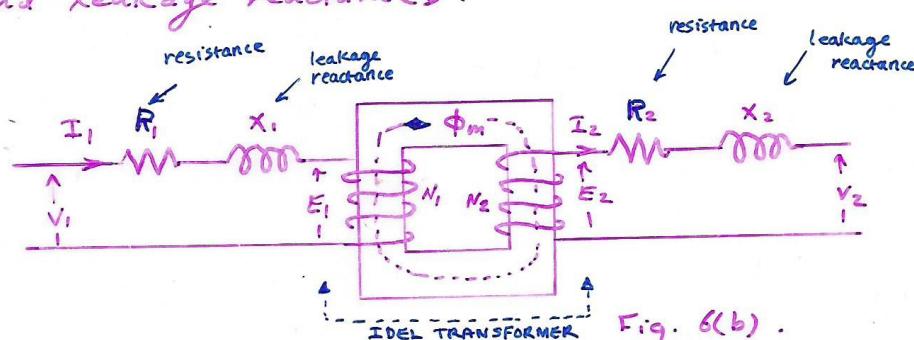


Fig. 6(b)

In Fig. (6) : $\frac{E_1}{E_2} = \frac{N_1}{N_2}$

Thus the actual transformer is equivalent to an ideal transformer plus external impedances. The exact composite equivalent circuit of an actual transformer is therefore can be drawn as shown in Fig. 7

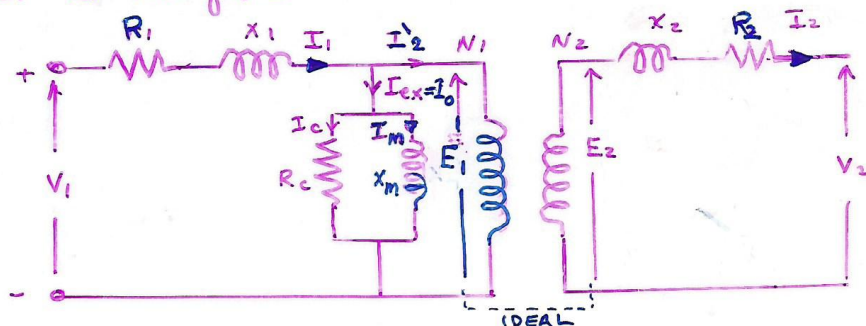
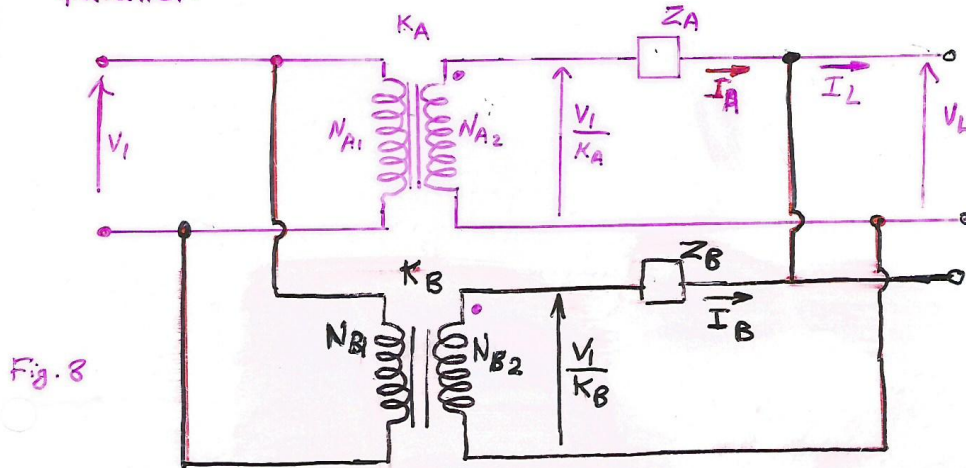


Fig. 7

3. Paralleling of Transformers

In Fig. 8 two transformers are shown connected in parallel.



Let their resistances and reactances referred to the secondary are, for the purposes of calculation, shown as impedances Z_A and Z_B . If the transformation ratios are $K_A = \frac{N_{A1}}{N_{A2}}$ and $K_B = \frac{N_{B1}}{N_{B2}}$, thus the primary voltage V_1 referred to the secondary sides of both transformers are V_1/K_A and V_1/K_B . The load current is

$$I_L = I_A + I_B \quad \therefore I_B = I_L - I_A$$

$$V_L = \frac{V_1}{K_A} - I_A Z_A \quad \& \quad V_L = \frac{V_1}{K_B} - (I_L - I_A) Z_B$$

Solving these two equations we get

$$I_A = \frac{Z_B I_L}{Z_A + Z_B} + \frac{V_1 (K_B - K_A)}{K_A K_B (Z_A + Z_B)}$$

$$I_B = \frac{Z_A I_L}{Z_A + Z_B} - \frac{V_1 (K_B - K_A)}{K_A K_B (Z_A + Z_B)}$$

$$\therefore I_{cir} = \frac{V_1 (K_B - K_A)}{K_A K_B (Z_A + Z_B)} \quad (\text{circulating current})$$

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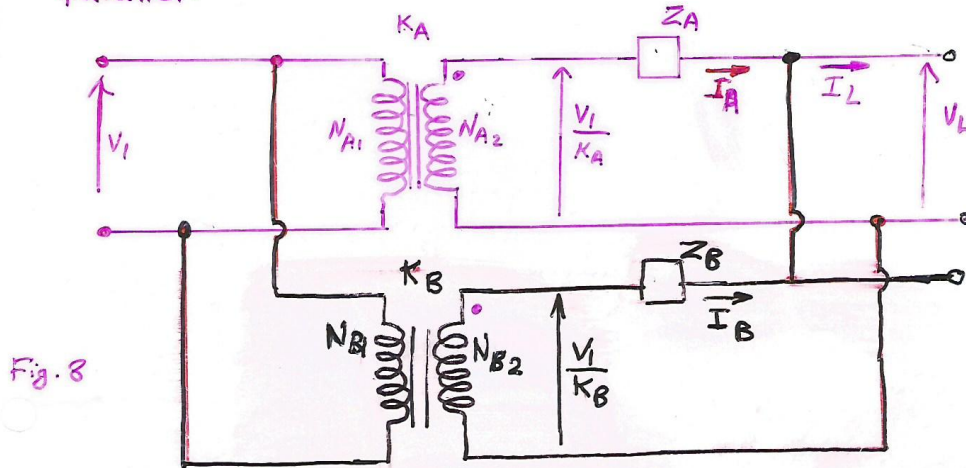


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$$I_L = I_A + I_B \quad \therefore I_B = I_L - I_A$$

$$V_L = \frac{V_1}{K_A} - I_A Z_A \quad \& \quad V_L = \frac{V_1}{K_B} - (I_L - I_A) Z_B$$

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If $K_A = K_B$

Transformers with equal turns ratio

$$\therefore I_{cir} = 0 \quad \& \quad \frac{I_A}{I_B} = -\frac{Z_B}{Z_A}$$

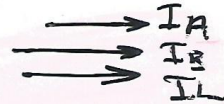
Note 1:

If the ratio $\frac{R_A}{X_A}$ and $\frac{R_B}{X_B}$ are equal then the currents I_A and I_B will be in phase, and we can write

$$I_L = I_A + I_B$$

$$\frac{I_A}{I_B} = \frac{Z_B}{Z_A}$$

$$\& \quad I_A = \frac{Z_B}{Z_A + Z_B} I_L \quad , \quad I_B = \frac{Z_A}{Z_A + Z_B} I_L$$



Note 2

$$\text{If } \frac{X_A}{R_A} \neq \frac{X_B}{R_B} \quad \text{i.e. } \theta_A = \tan^{-1} \frac{X_A}{R_A} \neq \theta_B = \tan^{-1} \frac{X_B}{R_B}$$

Then the currents are not in the same phases as shown in Fig. 9.

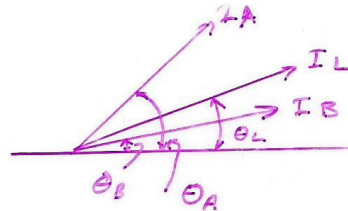


Fig. 9

Notes:

For ideal parallel operation:

1. The two transformers must have same polarity
2. Equal or nearly equal voltage ratings.
3. Equal % values of impedances.
4. Equal X/R ratio
5. Same frequency.

where percentage resistance and reactances are

$$\% R = \frac{100 I R}{V_n}$$

I = full load current
 V_n = normal voltage.

and

$$\% X = 100 X / V_n$$

The primary current I_1 can be resolved into two components, a load component and an exciting component. The load component I_1' is defined as the component current in the primary which would exactly counter act the mmf of the secondary current I_2 . Thus for opposing currents:

$$I_1' = \frac{N_2}{N_1} I_2.$$

It is equals the secondary current referred to the primary as in an ideal transformer.

No Load Vector diagram

The vector (phasor) diagram of Fig.7 will aid in summarizing the transformer no-load components that have been described previously. If the secondary of the transformer is open the vector diagram is as shown in Fig.8.

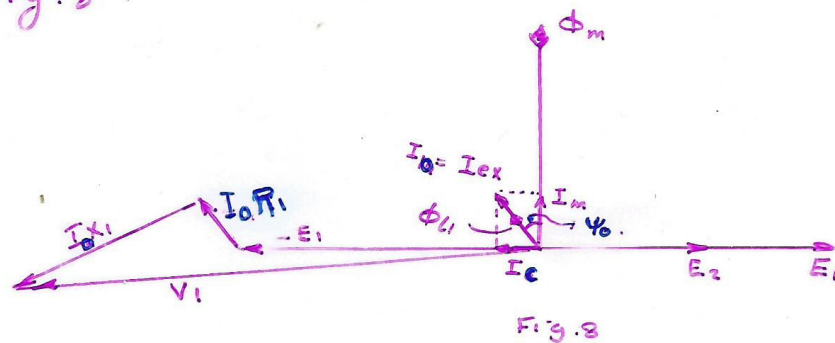


Fig.8

In this diagram E_2 is the voltage build up in the secondary winding. It caused by the mutual flux ϕ_m which is the same flux that builds up the counter e.m.f E_1 in the primary. A magnetizing current I_m is necessary to produce ϕ_m . Core losses require a component of current in phase with the voltage $-E_1$. This component $I_c (= I_h + e)$. This component added vectorially to I_m , results in the no load current I_0 . The primary leakage flux ϕ_{l1} reach its maximum at the same time as I_0 , consequently it is in time phase with I_0 as shown.

in fig.8. ϕ_{l1} builds up a voltage 90 degrees behind it which

is represented by its equivalent $I_0 X_L$ voltage drop. The resistance of the primary winding causes a voltage $I_0 R_1$, which is shown in phase with the primary no load current I_0 . The applied voltage V_1 is used up in overcoming the leakage reactance and primary resistance drop and the induced voltage $-E_1$ in the primary winding.

Vector diagram of the loaded transformer

The leakage fluxes ϕ_{l1} and ϕ_{l2} are proportional to the primary and secondary current I_1 and I_2 respectively. It is convenient to represent the voltage produced by these fluxes as reactance voltage drops and to assume that these voltage drops are produced by fixed values of leakage reactance X_1 and X_2 .

If we now assume that a load is connected to the secondary winding of the transformer shown in Fig.7, the vector diagram of the transformer will be constructed as follows:

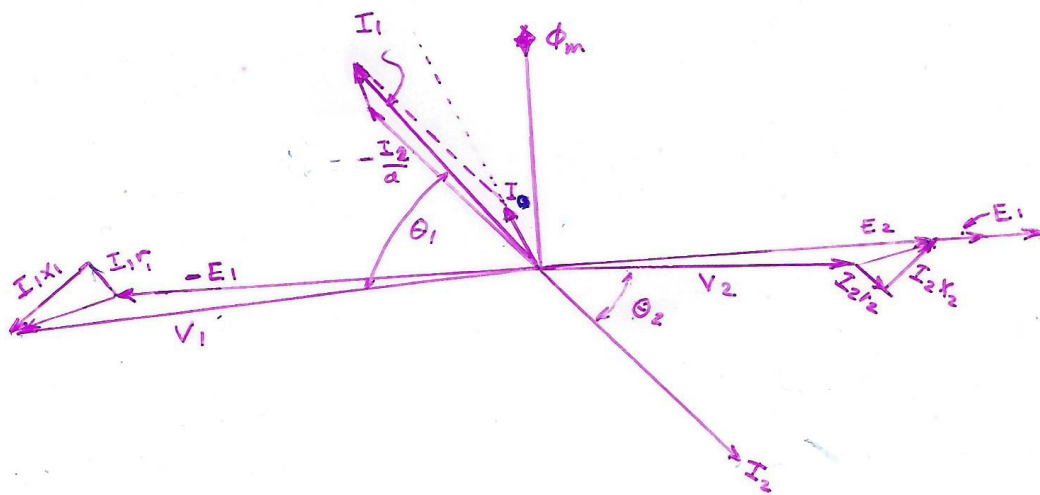


Fig.9 Transformer with lagging p.f.

For unity p-f load:

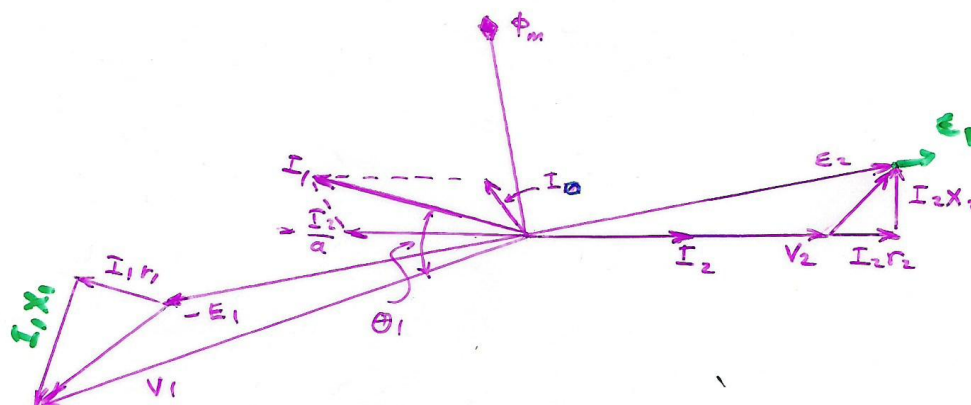


Fig.10 Unity p.f load $\theta_2 = 0^\circ$

For leading p-f load:

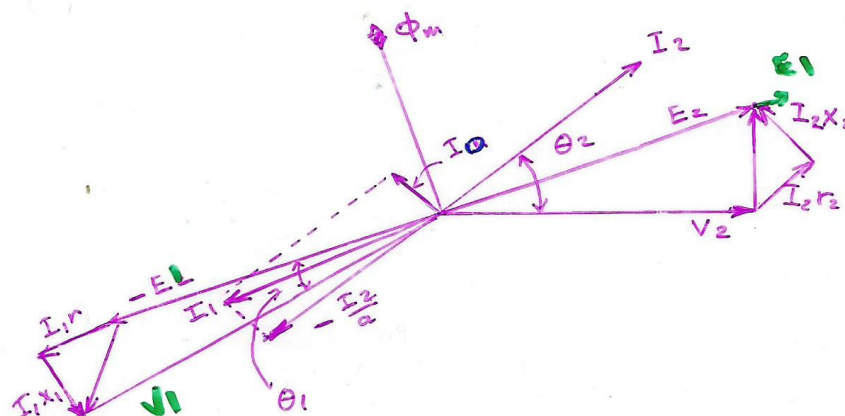


Fig.11. Leading p.f. load

Starting with V_2 (as a reference vector), I_2 is constructed both in magnitude and direction such that θ_2 corresponds to the power factor of the load. The $I_2 r_2$ vector is constructed so that it is in phase with the current I_2 and the vector $I_2 x_2$ is leading by 90° . The vector sum V_2 , $I_2 r_2$ and $I_2 x_2$ is E_2 the voltage induced in the secondary by the mutual flux ϕ_m . The voltage produced by ϕ_m on the primary winding E_1 is in time phase with E_2 . The ampere turns of the secondary $N_2 I_2$ must be canceled by the load component of the current in the primary I_2/a such that

$$N_1 \frac{I_2}{a} = I_2 N_2.$$

Since the induced voltages are both produced by ϕ_m ,

$$\therefore \frac{E_1}{E_2} = \frac{N_1}{N_2} = a.$$

The primary total current I_1 is obtained by adding the no-load current I_0 to $-I_2/a$ vectorially.

I_0 is constructed such that its magnitude is equal to the current taken by the primary when the secondary circuit is open. The angle of lag of I_0 behind $-E_1$ is assumed to be equal to $(90 - \psi_m)$ in Fig. 8.

(Note: In most power transformers the magnitude of I_0 is small compared with $-I_2/a$ (1-6%). Thus it can be neglected. Hence $\frac{I_1}{I_2} = \frac{1}{a}$).

The primary voltage V_1 is now determined by combining $I_1 r_1$ or $I_1 x_1$ vectors to $-E_1$ vector as shown in Fig. 9.

in Fig. 9.

This method of construction of the vector diagram can be applied for any condition of secondary load. Fig. 10 shows a vector diagram for unity p.f ($\theta_2 = 0^\circ$) and Fig. 11 shows a V.D for leading p.f load.

TRANSFORMER CLASSIFICATIONS

Because the wide range of applications of transformers in industry, they may be classified under the following categories:

A. General Application classification:

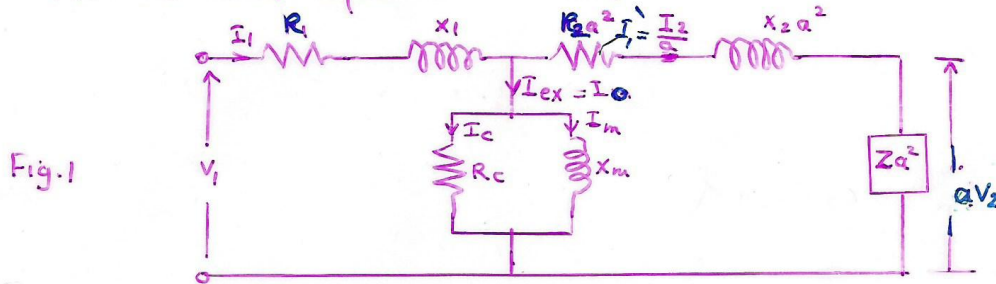
1. Power system transformer: transformer used in transmission and distribution systems.
2. Electronic transformers: transformers of many different types and applications used in electronic circuits.
3. Instrument transformers: transformers used in measurements of voltages or currents usually called voltage transformers or current transformers.

B. Classification by frequency ranges:

1. Power : 50, 60, or 400 Hz.
2. Audio : used in many communication circuits to operate at audio frequencies.
3. Ultra high frequency (UHF).
4. Wide-band: electronic transformers operating over a wide range of frequencies.
5. Narrow-band: electronics transformers designed for a specific frequency range.
6. Pulse: transformers designed for use with pulsed or chopped excitation, both in electronics and power system applications.

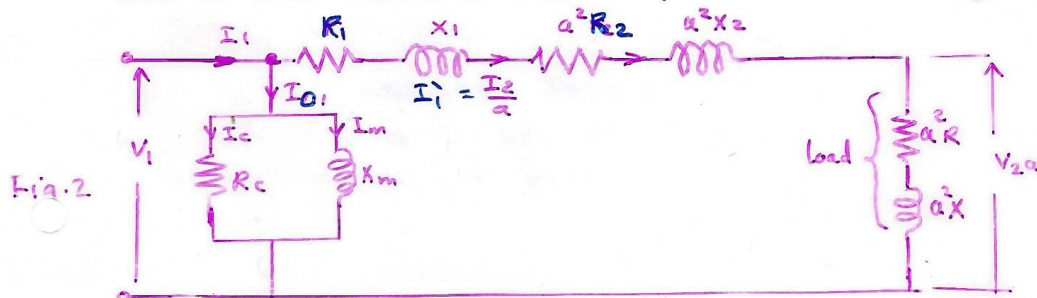
1. Equivalent Circuits : For purposes of calculation and design, it is often expedient to replace the actual transformer by an equivalent ckt that has similar properties.

1.1. The exact equivalent circuit

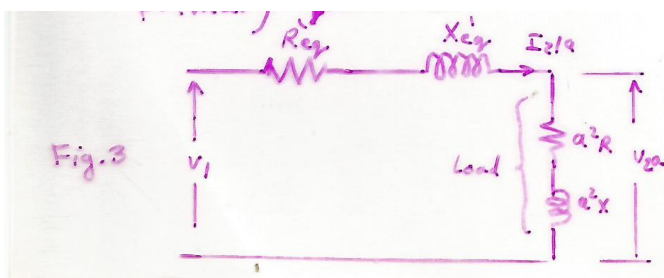


1.2. The approximate equivalent circuit

Because the no load current I_0 of transformers is small, it cause very little voltage drop through r_1 and X_1 . The exact equiv. ckt of Fig. 1. then can be simplified by connecting the no-load network across V_1 as shown in Fig. 2.



For studying the effect of load on the voltage, the circuit is frequently simplified to that of Fig. 3. Here the no-load branch (R_c & X_m) is dropped altogether. Hence Fig. 3. gives the equivalent circuit and its vector diagram in terms of primary quantities.

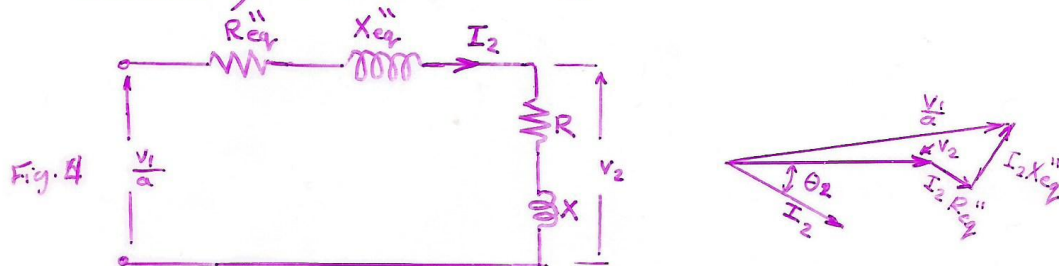


The following Substitutions were made:

Primary equivalent resistance	$R_{eq}' = R_1 + a^2 R_2$
Primary equivalent reactance	$X_{eq}' = X_1 + a^2 X_2$

Values referred to primary

The equivalent circuit with all impedances referred to the secondary is shown in Fig. 4.



where:

Secondary equivalent resistance	$R_{eq}'' = R_2 + \frac{R_1}{a^2}$
Secondary equivalent reactance	$X_{eq}'' = X_2 + \frac{X_1}{a^2}$

Values referred to secondary.

1.3. Voltage Regulation: Voltage regulation is defined as the ratio of the difference between the secondary voltage at no load and the secondary voltage on full (rated) load to V_2 at full load. On secondary side this is given as

$$\% \text{ Regulation} = \frac{V_{2 \text{ no load}} - V_{2 \text{ full load}}}{V_{2 \text{ full load}}} \times 100.$$

$$\text{or } \% \text{ Regulation} = \frac{V_1/a - V_{2 \text{ full load}}}{V_{2 \text{ full load}}} \times 100.$$

on the primary side:

$$\% \text{ Regulation} = \frac{V_1 - a V_2}{a V_2} \times 100$$

Example 4: A 150kVA transformer rated at 2400/240V has the following parameters

$$R_1 = 0.2 \Omega$$

$$X_1 = 0.45 \Omega$$

$$R_c = 10,000 \Omega$$

$$X_m = 1550 \Omega$$

$$R_2 = 0.002 \Omega$$

$$X_2 = 0.0045 \Omega$$

Use the exact equivalent circuit and refer the circuit to the primary side, calculate the voltage regulation of the transformer at rated load with 0.8 power factor lagging.

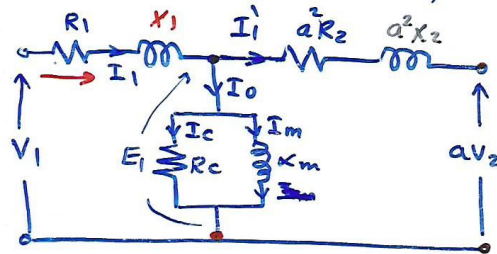
Sol. Use the exact circuit referred to primary

$$V_2 = 240$$

$$a = \frac{N_1}{N_2} = \frac{2400}{240} = 10$$

$$P.F. = \cos \theta = 0.8 \text{ lagging}$$

$$\therefore \theta = -36.8^\circ$$



$$\frac{KVA}{V_2} = I_2 = \frac{150 \times 10^3}{240} = 625 \text{ A} \quad (I_2 = \frac{KVA}{V_2} \times 10^3)$$

$$I_1' = \frac{N_2}{N_1} \times I_2 = \frac{I_2}{a} = \frac{625 \angle -36.8^\circ}{10} = 50 - j37.5 \text{ A}$$

$$aV_2 = 10 \times 240 \angle 0^\circ = 2400 \angle 0^\circ = 2400 + j0 \text{ V}$$

$$a^2R_2 = 0.2 \Omega \quad \& \quad a^2X_2 = 0.45 \Omega$$

$$E_1 = aV_2 + I_1' (a^2R_2 + j a^2X_2) = 2400 + j0 + (50 - j37.5)(0.2 + j0.45) \\ = 2427 + j15 = 2427 \angle 0.35^\circ \text{ V}$$

$$I_m = \frac{2427 \angle 0.35^\circ}{1550 \angle 90^\circ} = 1.56 \angle -89.65^\circ = 0.0095 - j1.56 \text{ A}$$

$$I_c = \frac{2427 \angle 0.35^\circ}{10,000} = \frac{2427 + j15}{10,000} \approx 0.2427 + j0 \text{ A}$$

$$I_0 = I_m + I_c = 0.25 - j1.56 \text{ A}$$

$$I_1 = I_0 + I_1' = 50.25 - j39.06 = 63.65 \angle -37.85^\circ$$

$$V_1 = E_1 + I_1 (R_1 + jX_1) = (2427 + j15) + (50.25 - j39.06)(0.2 + j0.45) \\ = 2455 + j30 = 2455 \angle 0.7^\circ \text{ V}$$

$$\therefore \% \text{Reg.} = \frac{V_1 - aV_2}{aV_2} \times 100 = \frac{2455 - 2400}{2400} \times 100 = \underline{\underline{2.3\%}}$$

Efficiency :-

A detailed analysis of the transformer losses will take into account the dielectric loss and stray load loss, but for the present analysis they will be assumed to be included in the iron loss and the copper loss respectively, the iron losses ΔP_{Fe} are independent of load current if the voltage and frequency are constant. So they constitute the fixed losses, the copper loss ΔP_{Cu} are $I_1^2 R_{eq}$ and $I_2^2 R_{eq}$.

$$\text{but } \eta = 1 - \frac{\text{losses}}{\text{output} + \text{losses}}$$

$$\text{Efficiency } \eta = \left(1 - \frac{\Delta P_{Cu} + \Delta P_{Fe}}{\text{Output} + \Delta P_{Cu} + \Delta P_{Fe}} \right) \times 100$$

$$\text{Output} = V_2 I_2 \cos \phi_2 .$$

The efficiency varies between 96% for small transformers and 99.5% for large transformers.

The efficiency is maximum when the Fixed losses = Variable losses, i.e.

$$\text{for } (\eta)_{\text{max}} \quad \Delta P_{Fe} = \Delta P_{Cu}$$

This can be obtained as follows:-

$$\text{Output} = V_2 I_2 \cos \phi_2$$

$$\text{Primary copper loss} = R_1 I_1^2$$

$$\text{secondary} \quad \quad \quad = R_2 I_2^2$$

$$\therefore \text{Total copper loss } \Delta P_{cu} = R_1 I_1^2 + R_2 I_2^2 = R_{2eq} I_2^2$$

Variable losses

$$\text{Iron loss} = \Delta P_{Fe} = \text{Constant.}$$

$$\therefore \eta = \frac{\text{output}}{\text{input}} = \frac{V_2 I_2 \cos \phi}{V_2 I_2 \cos \phi + R_{2eq} I_2^2 + \Delta P_{Fe}}$$

$$\text{for } \eta_{max}, \frac{d\eta}{dI_2} = 0.$$

$$\therefore R_{2eq} I_2 = \frac{\Delta P_{Fe}}{I_2}$$

$$\therefore I_2 = \sqrt{\frac{\Delta P_{Fe}}{R_{2eq}}}$$

All-day efficiency :-

A more suitable method of expressing the efficiency of transformer having a duty cycle over 24 hour period is on an energy basis. The output and losses are calculated in KWH over a 24 Hr day and the all day efficiency is defined as follows:-

$$\text{All day efficiency} = \frac{\text{Output in Killowatt-hour}}{\text{input in Killowatt-hour}}$$

in %

$$= \left(1 - \frac{\text{KWH losses}}{\text{Output (KWH)} + \text{KWH losses}} \right) \times 100$$

Example:-

A 20kW lighting transformer of ordinary efficiency 95 percent is on full load for 6 hr/day. Find the ~~overall~~ all-day efficiency if the full load losses are equally divided between copper and iron.

Solution:-

Total losses on full load = 5% of 20kW = 1kW
 $\therefore$ Iron losses = 0.5kW, and full load copper losses = 0.5kW.

output of 20 kW for 6 hr/day = 120 kWh

Copper loss of 0.5kW for 6 hr/day = 3 kWh

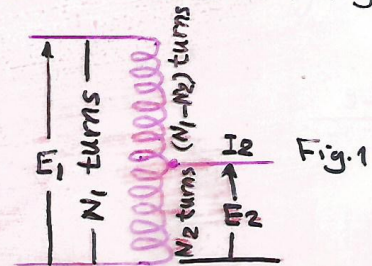
Iron loss of 0.5kW for 24 hr/day = 12 kWh

energy input during 24 hr = 135 kWh.

$\therefore$ All day efficiency = $\frac{120}{135} = 89\%$.

The Auto-transformers: A normal two-circuit transformer becomes an auto-transformer when the primary and secondary windings have a common part or parts as shown in Fig. 1. The secondary voltage being obtained by tapping off at any convenient point. As in the ordinary transformer, the transformation ratio is equal to the turn ratio

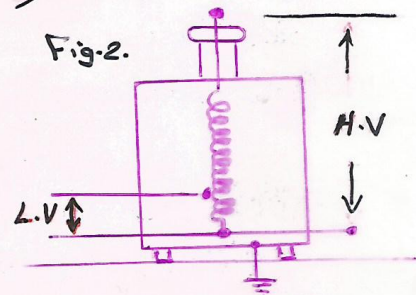
$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$



Note: For the same capacity and voltage ratio the auto-transformer requires less copper than ordinary transformer.

Note 2: Auto-transformers are commonly used in the production of small units generating H.V. for power system or testing applications. Fig. 2. shows an auto transformer energized from 220V. L.V. supply to produce H.V. (about 3000 V).

Notes: Two or more auto-transformers could be connected in chain to produce any required H.V. if necessary.



Three-Phase Transformers: The ordinary transformation of three-phase current at one voltage to three-phase current at another voltage can be achieved either by means of a single three-phase transformer or by three separate single-phase transformers as shown in Fig. 3. The latter (shown in Fig. 3(a)) may be called transformer bank.

There are many ways in which three-phase transformer winding can be connected. A few typical connections are shown in Fig. 4.

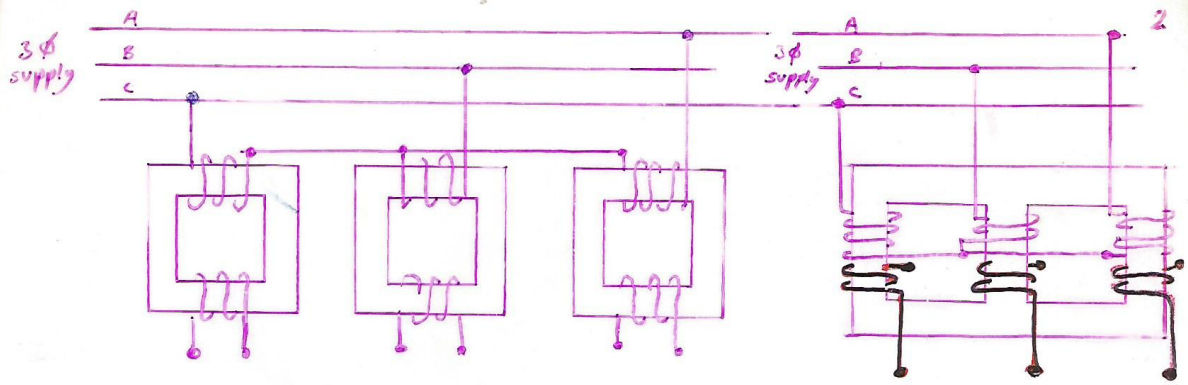
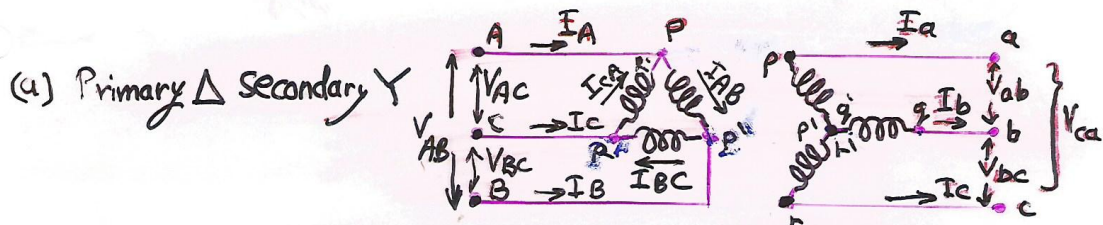


Fig. 3(a). Three-separate single-phase transformers

Fig. 3(b) Single three-phase transformer.



(b) Primary Δ secondary Δ



(c) Primary Y secondary Y

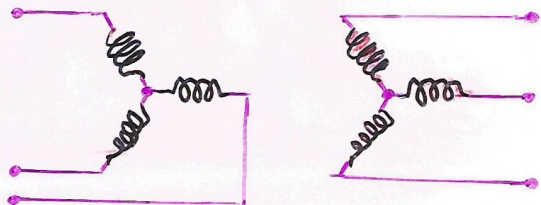


Fig. 4

In Fig. 4. (a) the windings labeled P-P' and p-p' constitute the primary and secondary windings respectively of one winding of the three-phase transformer. Similarly the other two windings are labeled Q-Q' and q-q' and R-R' and r-r'. It is useful to know the relationships between the voltages on the two sides of a three-phase transformer bank under sinusoidal steady-state condition:

3.2.1 Voltage Relationships

3

Assume a balanced 3 ϕ voltages V_{AB} , V_{BC} and V_{CA} are applied to the terminals of the delta connected side of the transformer bank. These voltages can be represented by phasors (Fig. 5.a). Since each winding on the delta connected side of the bank (primary side) is connected directly to the source, we have:

$$V_{AB} = V_{pp1}, \quad V_{BC} = V_{qq1} \quad \text{and} \quad V_{CA} = V_{rr1}.$$

These are called the primary voltages.

It is clear that the induced voltages on the Y-connected side (secondary side) are given by:

$$V_{pp1} = a V_{pp1} = a V_{AB}$$

$$V_{qq1} = a V_{qq1} = a V_{BC}$$

$$V_{rr1} = a V_{rr1} = a V_{CA}$$

where

$$a = \frac{N_{pp1}}{N_{pp1}} = \frac{N_{qq1}}{N_{qq1}} = \frac{N_{rr1}}{N_{rr1}} = \text{turns ratio / phase}$$

Since the transformers are assumed to be ideal, these secondary phase voltages are in phase with the corresponding primary phase voltages. In other words V_{pp1} and V_{AB} are in phase, V_{qq1} and V_{BC} are in phase, and V_{rr1} and V_{CA} are in phase.

The secondary phase voltages are shown in Fig. 5.b. The secondary line voltages V_{ab} , V_{bc} and V_{ca} are then obtained by addition of the appropriate phase voltages. From Fig. 4.a, it is clear that:

$$V_{ab} = V_{pp1} - V_{qq1} = \sqrt{3} a V_{AB} \angle 30^\circ$$

$$V_{bc} = V_{qq1} - V_{rr1} = \sqrt{3} a V_{BC} \angle 30^\circ$$

$$V_{ca} = V_{rr1} - V_{pp1} = \sqrt{3} a V_{CA} \angle 30^\circ$$

The secondary line voltages are also shown in Fig. 5.b.

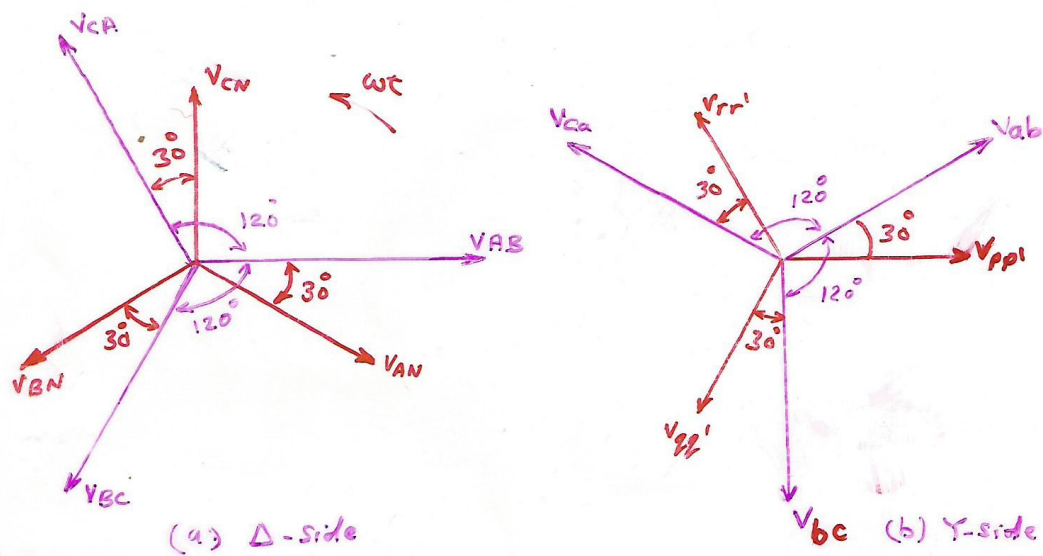
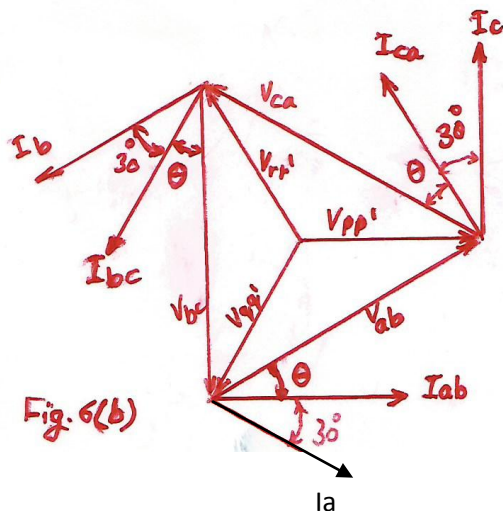
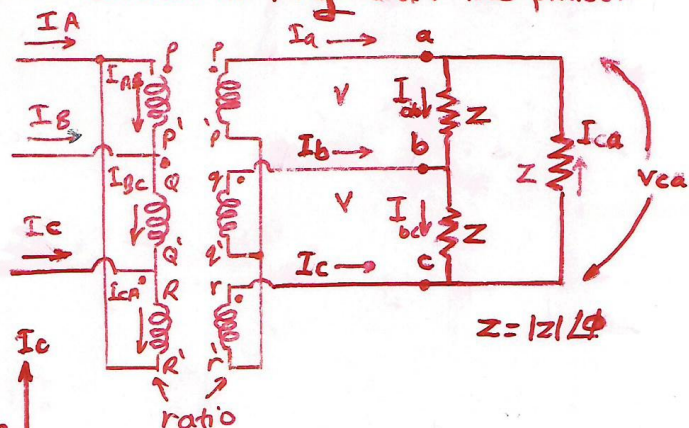


Fig. 5

Current Relationships Assume that a balanced delta-connected load is supplied by the three-phase Δ -Y transformer bank shown in Fig. 6a. The phasor

Fig. 6a



representation of the voltage and currents on the secondary side are shown in Fig. 6b. The line current are

$$\begin{aligned} I_a &= I_{ab} - I_{ca} \\ I_b &= I_{bc} - I_{ab} \\ I_c &= I_{ca} - I_{bc} \end{aligned}$$

Three-Phase transformer phase relationships

Phase Dis- place- ment.	Main Group No.	Vector Group Ref. No. & Symbol.	Marking of Line Terminals and Vector Diagram of Induced Voltages.		Winding Connections and Relative Position of Terminals.
			H.V. Winding.	L.V. Winding.	
Col. 1 0°	1	II Yy 0			
	2	21 Yy 6			
Col. 2 180°	3	31 Dy I			
	4	41 Dy II			

2. Special Considerations:

5

For high efficiency and good regulation the no load current I_0 must be small. i.e. I_m and I_c must be small. I_m is kept small by having a closed magnetizing circuit, i.e. one without airgap. I_c is kept small by using low-loss iron and not working at too high flux density. Grain-oriented cold-rolled sheet steel lamination gives important reduction in core losses and magnetizing m.m.f. The core is built up of laminations. These lamination reduces the eddy current losses.

2.1 Transformer losses

1. Eddy current and Hysteresis losses: These losses are calculated as follows:

(i) In order to calculate the eddy current loss imagine a portion of one core strip of thickness t cm, as shown in Fig. 5. Consider a rectangular prism of dimension $t \times t \times l$ m. The area of the front face is $t \times t = t^2$ m². and since the flux enters this face at right angles, the eddy currents will flow along paths equal to the long sides. Consider two such paths each of width dx and distance x from the centre line a-b.

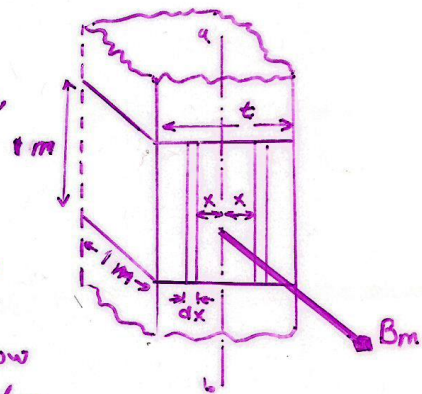


Fig. 5

Area of rectangle enclosed by paths = $2x$ m²

$\therefore$ Maximum flux entering the rectangle = $2x B_{max} \cdot lb$.

E.m.f induced in it = $4.44 \Phi_{max} N f = 4.44 (2x B_{max}) N f$

$= 4.44 (2x B_{max}) l f$ ($N=1$)

$\therefore E = 8.88 B_{max} f x$ Volts.

The current set up is confined to the two long

sides, the total resistance of the path being therefore 6

$$\rho \cdot \frac{l}{a} = \frac{2\rho}{dx \times 1} = \frac{2\rho}{dx}$$

Hence eddy-current loss in the two strips:

$$\frac{E^2}{R} = \frac{(8.88)^2 B_{\max}^2 f^2 x^2}{2\rho} \cdot dx$$

$\therefore$ Total eddy-current loss in the rectangular prism considered.

$$\begin{aligned} P_e &= \int_0^{t/2} \frac{(8.88)^2 B_{\max}^2 f^2}{2\rho} \times x^2 dx \\ &= \frac{(8.88)^2 B_{\max}^2 f^2}{48\rho} \times t^3 \end{aligned}$$

Now the volume of the prism is $t \times 1 \times 1 \text{ m}^3$. Hence, eddy current loss / m^3

$$= \frac{(8.88)^2 B_{\max}^2 f^2}{48\rho} \times t^2$$

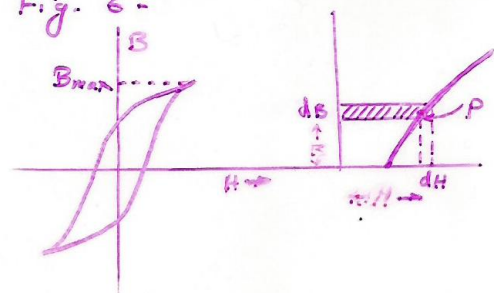
Thus in a transformer with a core of given volume and lamination thickness

$$\therefore \text{eddy current loss } P_e = K_e f^2 B_{\max}^2 \text{ Watts.}$$

Note: If f and $B_{\max}$ are constant, therefore $P_e \propto t^2$, this means that P_e depends on the square of the lamination thickness, so it is always possible to reduce this factor by choosing thinner laminations.

(ii) To estimate the Hysteresis loss, consider the B-H curve shown in Fig. 6.

At point P, when the magnetizing force is H and the flux density B , Let H increase by dH and B by dB .



Let

i = Current correspond to P ,

N = turns on the magnetizing coil,

A = Area of cross-section of the specimen

and L = magnetic path of the specimen.

$$\text{Total flux } \Phi = BA.$$

$$\text{Rate of change of flux} = \frac{d\Phi}{dt} = A \frac{dB}{dt}$$

Applied voltage required to cause the change at this rate:

$$V = AN \cdot \frac{dB}{dt}$$

Energy used in time dt ,

$$dW = Vi \cdot dt = AN i dB$$

$$= AL \left(\frac{IN}{L} \right) dB \text{ joules.}$$

$$= H dB \text{ joules per cubic meter}$$

$$= \text{area of the shaded strip in Fig. 7(b).}$$

In Fig. 7(a), we see the energy required as the current increases from zero to max. value to be the summation of the areas of the elemental strips. When the current is reduced to zero the energy returned to the circuit is again shown a shaded area Fig. 7(b)

and Finally the difference between the two is the loss of energy in the process (Fig. 7(c)). The energy loss in joules per cubic metre per one cycle is thus the area of the complete B/H loop. Hence for f cycle per seconds

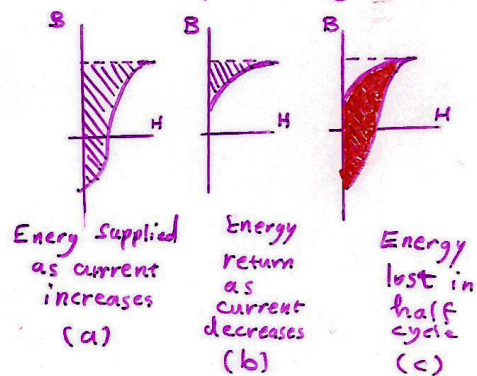


Fig. 7

$$P_h = (\text{area of loop}) f \cdot V \text{ Watts (V = volume)}$$

$$P_h = K_h B^x f V \text{ Watts. } \{x = 1.5 - 2.0 \text{ for iron}\}$$

Transformer Losses

2.1 CORE LOSSES

TRANSFORMER LOSSES

8

* eddy current + Hysteresis loss = $P_e + P_h$ Watts

This is called core loss P_c or iron loss P_i .

$$\therefore P_i = P_c = P_e + P_h = \text{constant}$$

2.2. Copper losses :

$$\text{Output in Watts} = V_2 I_2 \cos \theta_2$$

$$\text{Primary copper loss} = r_1 I_1^2$$

$$\text{Secondary copper loss} = r_2 I_2^2$$

$$\therefore \text{Total copper loss} = r_1 I_1^2 + r_2 I_2^2$$

2.3. Transformer Efficiency

$$\therefore \text{Efficiency} = \frac{\text{Output}}{\text{Intake}} = \frac{V_2 I_2 \cos \theta_2}{V_2 I_2 \cos \theta_2 + (r_1 I_1^2 + r_2 I_2^2) + P_i}$$

$$= \frac{V_2 \cos \theta_2}{V_2 \cos \theta_2 + I_2 \left(r_1 \left(\frac{I_1}{I_2} \right)^2 + r_2 \right) + \frac{P_i}{I_2}}$$

For a given p.f the efficiency is maximum when $I_2 \left[r_1 \left(\frac{I_1}{I_2} \right)^2 + r_2 \right] + \frac{P_i}{I_2}$ is minimum. Hence:

$$I_2 \left(\frac{r_1}{a^2} + r_2 \right) + \frac{P_i}{I_2} = R_{eq}'' I_2 + \frac{P_i}{I_2}$$

Now their product is a constant ($R_{eq}'' I_2 \times \frac{P_i}{I_2} = R_{eq}'' P_i$) and therefore their sum is a minimum when they are equal. Hence for max. efficiency:

$$R_{eq}'' I_2 = \frac{P_i}{I_2}$$

$$\therefore R_{eq}'' I_2^2 = P_i$$

$\therefore$ The efficiency is maximum when the copper loss equal to the iron (core) loss.

B-H Characteristics (Magnetization 4/5)

9

Magnetic circuit, in general, contains Ferromagnetic materials. The relation between B & H in these circuits are, more or less, non linear. The B - H curve is the results of "DOMAIN" changes within the magnetic material. If an external magnetizing force, H , is applied to the material, those domains that happen to be in line with the direction of H tend to grow, increasing B (region I in Fig. 8). In region II as H is further increased, the domains move rapidly until each crystal of the material is a single domain. In region III, the domains rotate in a direction until all domains are aligned with H . This results in magnetic saturation and the magnetic flux density B within the material cannot increase beyond the saturation density B_s .

- * This property of ferromagnetic material is called "SATURATION"

- * Another property of ferromagnetic material is: "HYSTERESIS";

Suppose now, the magnetic circuit material is subjected to cyclic magnetization, as shown in Fig. 9. The magnetic

intensity H is gradually increased by increasing the current in the excitation winding. The flux density B increases along the curve Oa , which is the same curve shown in Fig. 8. At a , $H = H_{max}$ and $B = B_{max}$. The magnetic intensity H is now gradually decreased. B will now vary along different curve ab . When H reduced to

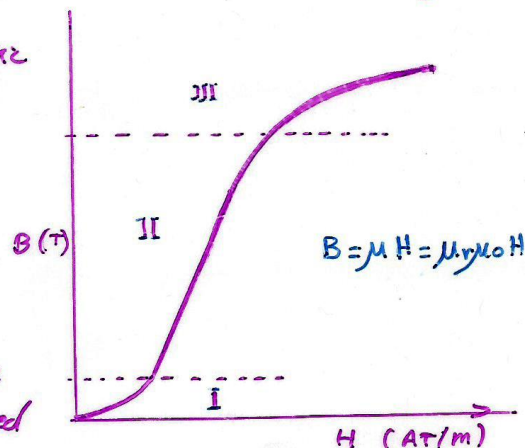


Fig. 8 A typical B - H curve

zero, $B = B_b$. This characteristic is known as hysteresis, and B_b is known as residual flux density.

If the current is now made to flow in the opposite direction and gradually increased, the magnetic material will be magnetized in the opposite direction.

B will vary along the curve bcd . At d , $H = -H_{max}$ and $B = -B_{max}$.

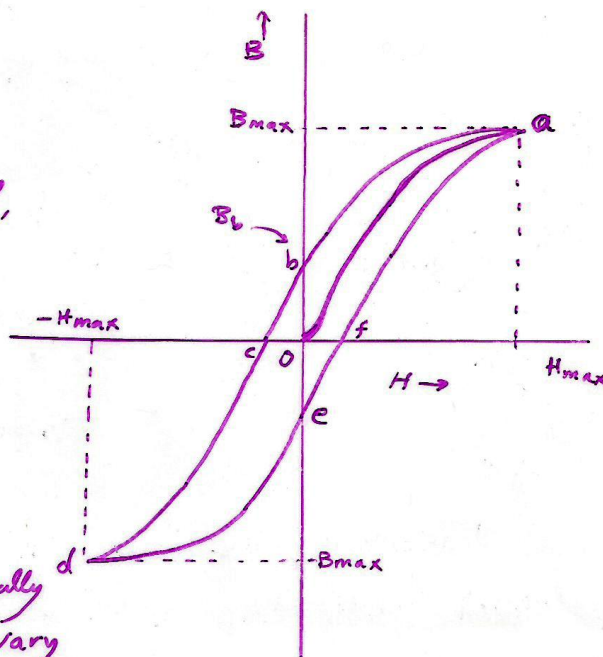


Fig. 9: A-C M.M.F and resultant Hyst. loop.

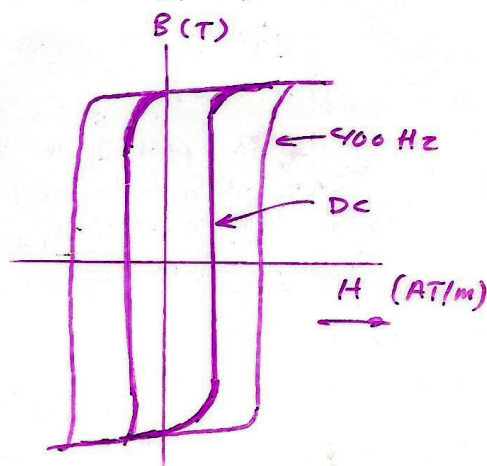
If H is now gradually decreased, B will vary along the curve $defa$ due to hysteresis. This closed loop is known as Hysteresis's loop. The area within the loop is related to the energy required to reverse the magnetic domain walls as the magnetizing force is reversed. This is a nonreversible energy and results in ^{an} energy loss known as the "Hysteresis Loss". This area varies with temperature and the frequency of reversal of H in a given material.

Hysteresis loss can also be calculated by Steinmetz formula in terms of energy loss:

$$W_h = \eta f (B_{max})^n \quad \text{Joules}$$

where η = Steinmetz coefficient which depends on the type of material

$$n \approx 1.5 - 2.5$$



In terms of power :

$$P_h = K_h f (B_{max})^n \quad \text{Watts}$$

Hysteresis loop for different frequencies for Deltaimax material.

where $K_h = \text{Constant}$.

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In Summary, if the mmf acting in a magnetic circuit is a.c., then the B-H Curve takes the form shown in Fig. 9. The loop shown is known as hysteresis loop. and the area within the loop is proportional to the energy loss (as heat) per cycle. This energy loss is known as HYSTERESIS LOSS.

In addition, eddy-current loss due to the eddy currents induced in the core material of a magnetic circuit excited by ac mmf. is another feature of an a.c. operated magnetic circuit. The losses due to hysteresis and eddy currents, collectively known as CORE LOSSES or IRON LOSSES, are approximately given

by:

$$\text{eddy-current loss : } P_e = K_e f^2 B_{\max}^2 \text{ W/Kg.}$$
$$\text{hysteresis loss : } P_h = K_h f B_{\max}^n \text{ W/Kg.}$$

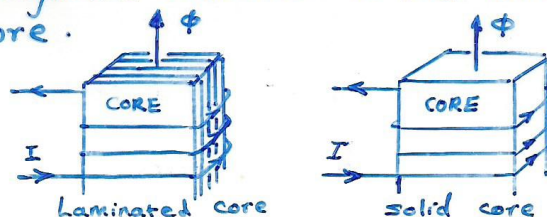
(n = 1.5 - 2.5).

where K_e is a constant ^{depends} on material conductivity and thickness, K_h is another constant, depending on the hysteresis loop of the material, and f is the frequency of excitation.

NOTES

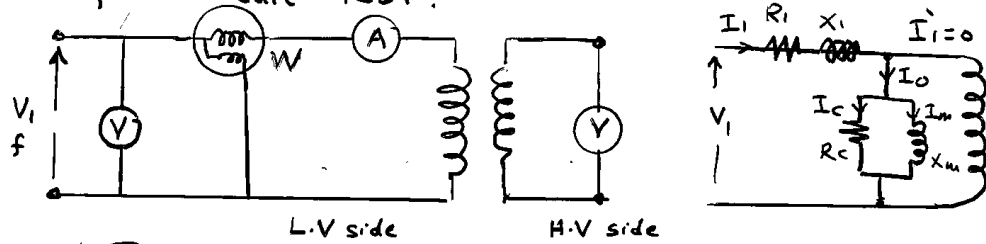
* Hysteresis losses may be reduced by using "good" quality electrical steel, having a narrow hysteresis loop, for the core material.

* To reduce eddy-current loss, the core is made of laminations, or thin sheets, with very thin layers of insulation between the laminations. Laminating a core increases its volume. hence a stacking factor is defined as the ratio of the volume actually occupied by the magnetic material to the total volume of the core.



TRANSFORMER TESTS

1. Open Circuit Test:



1. The H.V. side is open
2. input power supply is connected to L.V. side.
3. Using Wattmeter, Ammeter, Voltmeter for measuring core losses (iron losses), current I_0 and Voltage across the primary winding.

From meters readings:

$$\therefore I_1 = I_0 \quad , \quad W_0 = V_1 I_1 \cos \theta_0$$

$$\therefore \cos \theta_0 = \frac{W_0}{I_1 V_1}$$

$$\text{Hence } I_c = I_0 \cos \theta_0 \quad , \quad I_m = I_0 \sin \theta_0$$

$$W_0 = V_1 I_0 \cos \theta_0 = I_c^2 R_c \quad \therefore R_c = \frac{W_0}{I_c^2}$$

$$Z_m = \frac{V_1}{I_0} \quad \therefore X_m = \sqrt{Z_m^2 - R_c^2}$$

$$\text{or } X_m = V_1 / I_m$$

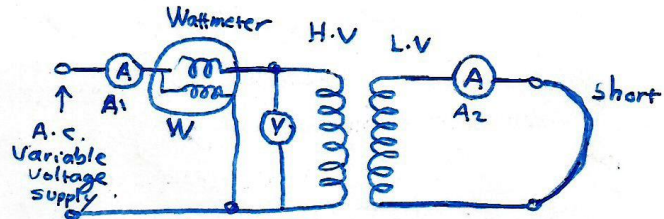
$$\text{The turns ratio: } a \approx \frac{V_1}{V_2}$$

Hence from O.C. test we can find:

$$\begin{bmatrix} I_c, I_m \\ R_c, Z_m \\ X_m \\ \& \text{ } \end{bmatrix}$$

2. Short Circuit Test :

Low voltage (L.V.) side is short circuited.



On the primary side the reading of the wattmeter will give $I^2 R$ loss, hence:

$$W_{s.c.1} = I_{s.c.1}^2 \cdot R_{eq1}$$

, $I_{s.c.1}$ the short circuit current read by ammeter A_1 .

$$V_{s.c.1} = I_{s.c.1} Z_{s.c.1}$$

$$Z_{s.c.1} = \frac{V_{s.c.1}}{I_{s.c.1}}$$

$V_{s.c.}$ is the short circuit voltage read by voltmeter V .

$$\therefore R_{eq1} = \frac{W_{s.c.1}}{I_{s.c.1}^2}$$

$$X_{eq1} = \sqrt{Z_{s.c.1}^2 - R_{eq1}^2} = \sqrt{\left(\frac{V_{s.c.1}}{I_{s.c.1}}\right)^2 - R_{eq1}^2} \dots (1)$$

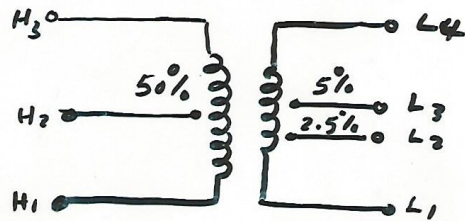
N.B In this test one winding (usually the L.V. Wdg) is short circuited and a reduced voltage is applied to the other winding to produce rated current in the short circuited winding.

Once R_{eq} & X_{eq} are obtained we can calculate x_1 and x_2 as follows:

1. R_1 can be measured experimentally by ammeter voltmeter method.
2. R_2 can be found : $R_1 + a^2 R_2 = R_{eq1}$
3. X_{eq1} can be found from eq (1).
4. It is usually assumed that :

$$x_1 = a^2 x_2 = \frac{1}{2} X_{eq1}$$

Tapped transformer



Tapped transformer.

Transformer polarity :-

